**§11. Shock Adiabat (Hugoniot Adiabat)**

The concept of *shock adiabat*, also known as *Hugoniot adiabat*, is widely used in physics because the change of state of a particle passing through a shock layer is usually adiabatic (see remark at the end of §10).

***Definition.*** An equation that determines the state functions behind a shock when those ahead of the shock are given is called a *shock adiabat*, or a *Hugoniot adiabat*, or a *shock Hugoniot*; the corresponding graph is also referred to as a *Hugoniot curve*.

**The Hugoniot adiabat for a perfect gas.** Consider a one-dimensional normal shock in a perfect gas, whose state on both sides is characterized by the equations (see (4.2.6)–(4.2.8) below)

*р* = ρ *R Т*, *u = c*v *T* =  (*R* , *c*v , γ - const). (3.11.1)

Suppose the effects of heat conduction (see (3.10.10)) and viscosity are negligible on both sides of the shock. The latter assumption implies that the stress tensor is isotropic and is determined by pressure:

 = − *р*(1)δ*ij*, = − *р*(2)δ*ij*. (3.11.2)

The balance equations for mass (3.10.4), momentum (3.10.6), energy (3.10.7), and entropy (3.10.8) across a normal shock written in the shock-fixed coordinates ( ) are similar to Eqs. (3.9.29)–(3.9.31) and (3.9.33):

**,** (3.11.3)

, (3.11.4)

**,** (3.11.5)

**,  =  >** 0. (3.11.6)

By eliminating velocities, the jump in internal energy across a normal shock is expressed as

**.**  (3.11.7)

This equation, known as the *Hugoniot equation*, can be combined with the expression for internal energy *u* in (3.11.1) to relate the pressure and density ratios across the shock:

.

Rewritten as density ratio expressed in terms of pressure ratio, or vice versa, this relation is the *shock adiabat* (*shock Hugoniot*) for a perfect gas:

**** or **** . (3.11.8)

The corresponding expression for *shock propagation speed*, derived from (3.11.3) and (3.11.4), is

 = − . (3.11.9)

At point *1* (initial state), the Hugoniot curve defined by Eq. (3.11.8) is second-order tangent to the *Poisson adiabat*, or *isentrope*, also known as *Poisson's law* (see (5.3.6) below),

; (3.11.10)

i.e., both  and  are equal for the curves. For a weak shock (with  ≡  − 1 << 1), the density jump corresponds to Poisson's law

 − 1 = , (3.11.11)

and the shock propagation speed is nearly equal to that of weak disturbances (sound) ahead of the shock wave (see Eq. (4.11.16) below):

 =   . (3.11.12)

An extremely strong compression shock (with ****>> , or  ) has the highest density ratio predicted by the Hugoniot equation for an adiabatic normal shock wave:

****= **** . (3.11.13)

For diatomic gases (e.g., air, which is a mixture of nitrogen N2 and oxygen O2), the adiabatic index is γ =  and ****= 6; for monatomic gases, γ =  and ****= 4.

Note that the density ratio across a shock predicted by Hugoniot equation (3.11.8) is strictly limited, whereas   as  ≡   in quasi-static adiabatic (isentropic) compression in accordance with Poisson's law.



*2*

*1*

1

1

**Fig. 3.11.1**. Hugoniot curve (bold) and Poisson adiabat (thin curve) in the plane (). The circle represents initial state *1*.

A convenient measure of the strength of a shock wave is the shock Mach number **M** defined as the ratio of shock propagation speed to the speed of sound ahead of the wave:

**M** = . (3.11.14)

The shock Mach number is related to the pressure change across a shock wave as follows:

, . (3.11.15)

Note that temperature increases across a shock when ****>>  and  >> . When it reaches *Т* ~ 103 K, the gas in the shock layer dissociates and ionizes. As a result, the adiabatic index  behind the shock increases, approaching γ = . It can be shown that  should be used in (3.11.10) under these conditions, which would even further restrict the maximum density ratio across the shock.

In condensed matter, the maximum shock compression ratio **** is much lower than in gases because compression is counteracted by intermolecular repulsive forces (see Chapter 5, §9).

*t*

B

С1

С2

С3

С*n*

С*n*+1

A1

*r*

A

О

S

A

О

**Fig. 3.11.2.** Schematic of spherically symmetric shock compression of a gas in the (*r*, *t*) plane, where *r* is radius and *t* is time.

Although shock compression ratio is limited, a gas can be compressed by a factor of thousands by multiple shocks. Figure 3.11.2 schematizes such a shock-compression process in the volume bounded by sphere А centered at point О. The surrounding layer is driven inwards by acoustic or blast waves or by multiple laser pulses focused onto the sphere. The spherical boundary rapidly accelerates and acts as a piston pushing the gas ahead of it towards the center. As a result, a converging shock wave S is initiated inside the sphere. Its trajectory is represented by curve АС1. The shock gains strength as it converges towards the center, bounces off the center at point С1, propagates back outwards, and bounces off the boundary at point А1. This sequence repeats itself so long as the inward-accelerated layer keeps acting as a piston and reflects shock waves. The waves pass through each particle of the gas many times, and the gas density increases several times with each shock passage.

At nuclear research centers in the USA (Livermore), France (Le Barp, near Bordeaux), and Russia (Sarov), ultrahigh-power laser pulses (with *Е*L ~ 106 J per target in 10–8 s, i.e. power of  ~ 1014 W) are projected to achieve a density of hydrogen isotopes (deuterium and tritium) as high as 103 g/cm3 and temperatures of up to 108 K for an initial density of ~10−4 g/cm3, thereby initiating a thermonuclear reaction producing a huge energy release (~1011 J per gram of hydrogen isotopes). Spherical targets 5 mm diameter are to be used. The facility for converting electricity to laser light (with efficiency of around 3%) and laser drivers (approximately 200 in total), together with energy storage and conversion systems, occupy an area of about 10,000 m2.

See also discussion of inertial collapse of a spherical gas bubble at the end of Chapter 4.

The Hugoniot curve lies above the Poisson adiabat (see Fig. 3.11.1) because part of the kinetic energy of the gas passing through the shock, equal to

****, (3.11.16)

is converted into internal energy, causing an additional increase in temperature compared to isentropic compression. Combining expression (5.3.4) for the entropy of a perfect gas (see Chapter 5) with shock Hugoniot (3.11.8) or (3.11.15) yields

**= ln  = ****

= . (3.11.17)

At points lying above the Poisson adiabat, including those on the Hugoniot curve, it holds that

 > 1,  > 1,  , (3.11.18)

and entropy increases across the shock (s(2) > *s*(1)) in accordance with entropy balance (3.11.6) when the shock is adiabatic (see (3.10.10)). Furthermore, it follows from (3.11.17) that the increase in entropy is a third-order quantity for a weak compression shock (with  << 1), which propagates at the speed of sound according to (3.11.12):

**. (3.11.19)

Thus, compression by a weak shock wave is a nearly isentropic process described by Poisson's law (3.11.11).

For a strong shock wave ( >> 1), with maximum compression ratio (3.11.13), it follows from (3.11.17) that the entropy increase asymptotically behaves as follows:

******. (3.11.20)

For a rarefaction shock,

 < 1,  < 1, , (3.11.21)

and (3.11.17) implies a decrease in entropy (*s*(2) < *s*(1)), which contradicts the second law of thermodynamics since entropy cannot decrease in an adiabatic process (see (3.11.6) and Chapter 5, §14). This leads to the following theorem.

***Zemplén Theorem*.** In a perfect gas, compression shocks are possible and rarefaction shocks are impossible.

The part of the Hugoniot adiabat corresponding to (unphysical) rarefaction shocks is represented by a dashed curve in Fig. 3.11.1.

The theorem on the impossibility of rarefaction shocks applies to any medium for which isentropes *s*(2) = const are concave in the (ϑ, *p*) plane:

 (3.11.22)

Convex isentropes (with  < 0), which make rarefaction shocks possible and compression shocks impossible, are characteristic of media undergoing phase transitions and of certain rubberlike materials.

Note the following essential difference between the Poisson and Hugoniot adiabats. A quasi-static isentropic compression or expansion is a change of state along the Poisson isentrope. The Hugoniot curve is the locus of all attainable final states, not the curve along which the system undergoing shock compression evolves from initial state *1* to final state *2*. It can be shown that compression by a steady shock is a process where the state of the system follows the line connecting points *1* and *2* in the  plane, referred to as the *Mikhel'son line* or the *Rayleigh line* (see Fig. 3.11.1).